Scaling of Impact-Loaded Carbon-Fiber Composites

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The application of scaling laws to fiber composite laminates is discussed. Particular emphasis is placed on the case of impact loading. Scaling difficulties and conflicts are identified and illustrated in an experimental program based on impacted carbon-fiber composite beams. It is shown that the lay-up of laminates is important in assessing the likely validity of scale-model tests for such composites. It is also observed that significant size effects may dominate strength modeling.

Nomenclature

a	= crack half-length
b	= beam width
E	= beam modulus
E_i	= impactor modulus
E_{11},E_{22}	= lamina moduli
F	= impact force
F_o	= reference impact force
G_{12}	= lamina shear modulus
h	= beam thickness
K_Q	= critical stress intensity factor
ℓ	= beam length
R_i	= impactor radius
s	= scale factor
t	= time
t_o	= reference time
V_i	= impactor velocity
V_o	= reference velocity
W	= plate width
Y	= compliance function
ϵ	= strain
$\dot{\epsilon}$	= strain rate
ν	= beam Poisson ratio
ν_{12}	= lamina Poisson ratio
ρ	= beam mass density
ρ_i	= impactor mass density
σ	= stress
σ_o	= yield stress
σ_u	= ultimate stress
τ	= material strain rate parameter
ω	= beam deflection

I. Introduction

ADVANCED fiber composite laminates offer significant strength and stiffness improvements over conventional metals on a weight-for-weight basis. These superior properties have resulted in a widespread use of such materials in secondary aerospace components and an increasing interest in primary structural applications.[1-3]

In the absence of the broad design base available for metal components, composite prototypes require extensive design evaluation, often through destructive testing. Such testing of prototypes is expensive, so that there is growing interest in the use of structural scale-model testing and the application of the principles of similitude to the design evaluation of fiber composite components. These principles have long played an important role in aerodynamic design and are being applied increasingly to complex structural problems.[4-6]

In this study, the application of the principles of similitude to dynamically loaded fiber composite laminated structures is discussed. General problems of scale-model testing are reviewed and specific ones, related to laminated composite components, are highlighted. An experimental program that illustrates scale-model testing of transversely impacted carbon-fiber composite beams is described.

II. Principles of Similitude

Goodier[7] has presented a detailed study of the application of the principles of similitude (dimensional anlaysis) to scale-model testing in structural engineering. More recently, Duffey et al.[8] have described the development of scaling laws for impact-loaded structures. As yet, special considerations for structures made of composite materials have not been reported.

In the most general form, the technique consists of identifying the relevant parameters associated with the problem in question. The dimensions of these parameters are listed and Buckingham's Pi-Theorem[9] is applied to obtain a complete set of nondimensional groups (Pi-terms). It should be noted that neither the set of dimensions nor the Pi-terms are unique. In the preparation of scale models, attempts are made to ensure that the values of the Pi-terms in model and prototype are identical. In general, however, this may be impossible.

As an illustration, consider the case of a transversely impacted beam. For the moment, suppose that the beam is made of a homogeneous, isotropic material and let the parameter of interest (the output parameter) be the central deflection $\omega(t)$. List the parameters as: 1) geometrical parameters ℓ, h, b; 2) material properties ν, E, ρ; 3) input parameters ν_i, E_i, ρ_i, R_i, V_i. Choose the fundamental dimensions as M, L, T. Applying the Pi-Theorem we have 13 parameters, and we may see that the rank of the dimensional matrix is 3, so that we need to form 10 Pi-terms. One set could be

$$\pi_1 = \omega/h \qquad \pi_6 = \nu$$

$$\pi_2 = \ell/h \qquad \pi_7 = \nu_i$$

$$\pi_3 = b/h \qquad \pi_8 = \rho_i/\rho$$

$$\pi_4 = R_i/h \qquad \pi_9 = \rho_i V_i^2/E$$

$$\pi_5 = E_i/E \qquad \pi_{10} = tV_i/h$$

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If the above represents a complete set, then we may write

$$f(\pi_1,\pi_2,\ldots\pi_{10})=0$$

Now consider a model in which the linear dimensions are scaled by a factor s with respect to the prototype. The terms $\pi_1-\pi_4$ are then the same in the model and prototype. If the same materials are used in the model m and prototype p, the terms $\pi_5-\pi_8$ are also scaled. Examine now the condition for π_9.

For prototype:

$$\pi_9=\left(\frac{\rho_i V_i^2}{E}\right)_p$$

For model:

$$\pi_9=\left(\frac{\rho_i V_i^2}{E}\right)_m$$

Thus, for scaling, the impactor velocity must be the same in the model and prototype. Finally, consider π_{10}. We require that

$$\left(\frac{tV_i}{h}\right)_m=\left(\frac{tV_i}{sh}\right)_p$$

Thus, time must be scaled in the model. That is, events in a subscale model ($s<1$) appear to occur faster than in the prototype. As an illustration, if the beam example above reaches full deflection after 10 ms, then a one-tenth-scale model ($s=0.1$) will reach its maximum deflection after 1 ms.

In this example, it appears possible to make an exact scale model. It is required, merely, to ensure that terms $\pi_1-\pi_{10}$ are the same in model and prototype. This is indeed the case if the 13 parameters above represent a complete set. However, consider the case where the beam is made from a material that is rate sensitive—that is, the stiffness, flow stress, and/or strengths may be dependent on the rate of loading. It is well known, for example, that mild-steels increase in yield strength and hardening rate as the strain rate increases. Suppose that some rate-dependent characteristic is described by the material rate law $\sigma=E(\epsilon+\tau\dot{\epsilon})$. Then, in defining the complete set of parameters an additional term τ, which reflects the material rate sensitivity, should be included. Of course, another Pi-term $\pi_{11}=t/\tau$ results.

Now, if the same material is used in the model and prototype as previously assumed, for similarity,

$$\pi_{11}=\left(\frac{t}{\tau}\right)_m=\left(\frac{t}{\tau}\right)_p$$

or

$$(t)_m=(t)_p$$

but the condition of similarity for π_{10} was that ($t_m=st_p$). There is a conflict. It is not, then, possible to produce an exact scale model for the rate-sensitive material. If the model were produced and the condition of π_{11} were relaxed, then strain rates in the model would be greater than in the prototype by a factor of $1/s$. Practically, this would give rise to apparently lower ductility in the model than in the prototype. It may even cause brittle behavior in the model when the prototype was ductile.

The above observations lead directly to a consideration of fracture phenomena. Jones[4] provides a discussion of a potential scaling conflict in such cases. An illustration is provided by the case of a thin rectangular plate of width W, containing a central crack of length $2a$. Under a uniform tensile stress normal to the crack, an ideal elastic, perfectly plastic material would result in an ultimate stress given by

$$\frac{\sigma_u}{\sigma_o}=\left(1-\frac{2a}{W}\right) \tag{1}$$

The terms (σ_u/σ_o) and (a/W) may be regarded as Pi-terms. Components made of such materials can be scaled for strength. Such materials are termed "notch-insensitive."

In linear elastic fracture mechanics, a brittle or notch-sensitive plate material would fail when

$$\frac{\sigma_u}{\sigma_o}=\frac{1}{Y(a/W)\sqrt{\pi(a/W)}}\times\left(\frac{K_Q}{\sigma_o\sqrt{W}}\right) \tag{2}$$

An additional Pi-term ($K_Q/\sigma_o\sqrt{W}$) appears in Eq. (2).

If K_Q and σ_o are material properties, then the strength of the cracked plate then depends on the absolute size. Thus, it is not sufficient to have identical dimension ratios (e.g., a/W) and the same materials to model the fracture behavior of notch-sensitive materials. Scaling also requires that ($K_Q/\sigma_o\sqrt{W}$) must be the same in the model and prototype. Again this may prove pratically impossible. Figure 1 illustrates this size effect for fracture.

III. Scaling of Laminated Fiber Composites

Section II reviewed the application of the principles of similitude to the problem of scale-model testing of dynamically loaded structures. It was shown that, generally, complete similarity is not possible, and that relaxation of some of the similarity requirements has to be made. Particular problems arise when dealing with rate-sensitive and/or notch-sensitive materials. Given the need for scale-model testing of composite subcomponents, consideration will now be given to scaling

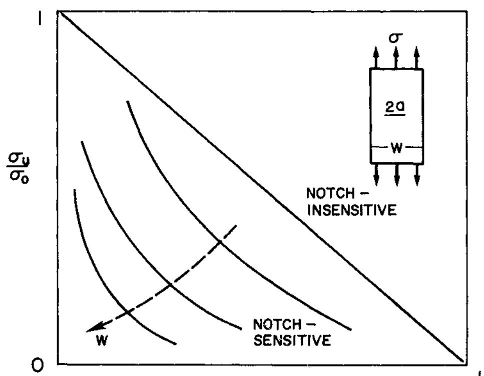


Fig. 1 Strength of notch-sensitive and notch-insensitive plates.

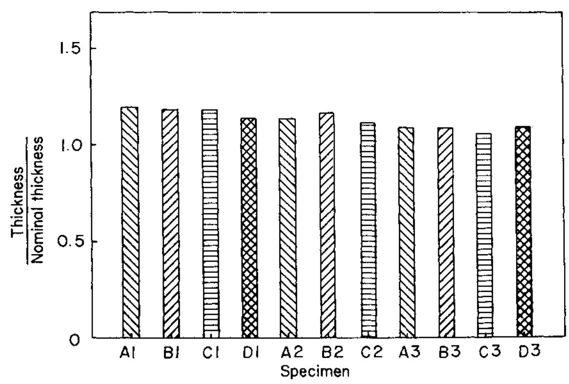


Fig. 2 Thickness variations for the different lay-ups and scale sizes.

laws for laminated fiber composites. Practically, such materials will consist of thin fibers (such as carbon, Kevlar, or glass), with diameters of about 0.00015–0.0005 in. (6–20 μm), in a plastic matrix. Individual laminae have a thickness of about 0.005 in. (0.13 mm). The most common matrix materials are epoxy resins, however, increasing consideration is being given to the use of thermoplastics.

Ideally, scaling of a composite prototype should include scaling of the microstructure; that is, the laminae thickness and the fiber diameter should also be scaled. This is, clearly, practically impossible. Of course, even in metallic structures, scaling of the microstructure is not attempted. In laminated composites, however, there are frequent interactions of micro- and macrostructural properties. Composites differ from metals in that they show a number of different damage mechanisms, including fiber fracture, delamination, and matrix cracking. All start on a microscopic scale and eventually interact with the macroscopic scale on a laminate level. One common sequence of events is the occurrence of a matrix microcrack that grows across a lamina until it reaches the interlaminar boundary. Thus, either delamination takes place and/or fiber fracture occurs.

On a larger scale, the notch-sensitivity of laminated composites strongly depends on the lay-up of the individual laminas. For example, a unidirectional laminate loaded in tension in the fiber direction is notch-insensitive. Since failure of ±45 deg laminates can occur without fiber fracture, similar behavior could be apparent, depending on the matrix shear characteristics. However, quasiisotropic laminates are known to be notch-sensitive. Thus, on both the microscopic and the macroscopic scales, significant scaling conflicts may arise in components made from laminated fiber composites. Given the varied and complex nature of the damage processes, it is not possible, a priori, to assess their effect on scale modeling.

Returning to the problem of rate-sensitivity, the behavior of laminated fiber composites is complex. Of the common fiber types, glass and Kevlar fibers are highly rate-sensitive, but carbon is rate-insensitive. To some extent, the epoxy resin matrix materials are rate-sensitive and thermoplastic materials more so. The degree of sensitivity shown by a composite clearly depends on the lay-up. Thus, the rate conflicts mentioned in Sec. II will be apparent in some laminated components. Also, the rate of loading may effect the nature of the interaction of the damage micromechanisms. It may then be concluded that the scale modeling of laminated composite components should be done with exteme caution, and that substantial testing is required to establish guidelines as to the pertinent scaling parameters.

IV. Scaled Impact Tests

Experimental Program

As a simple test case in the scaling of dynamically loaded composites, the example of the transversely impacted beam was taken. The beam was supported on rollers and impacted centrally by a free-falling mass. The impactor nose radius was made equal to the roller radius. The dimensions of the beams and the distance between the supports were made to minimize the effects of local indentations (of a Hertzian nature). This is not essential in the scaling arguments and steps were taken to ensure that these effects were scaled in the models. These design considerations did, however, simplify the analyses.

The laminated composite beams were fabricated from unidirectional pre-preg, which consisted of carbon fibers (AS4) in an epoxy matrix (3502). The lay-ups are shown in Table 1. The scaled laminates were produced with $n=1$, 2, and 3. For lay-up D, values of $n=1$ and 2 were produced (8- and 16-ply). Now, using values $n=1$, 1.5, and 2, the beam lengths and widths were $2.8n$ and $0.5n$ in. respectively. The rollers and impactor nose radii were $0.5n$ in. and the distances between rollers was $2.0n$ in. The impactors were cylindrical steel rods with rounded ends. Two sets of impactor masses were produced with nominal masses $0.33n^3$ lb ($150n^3$g) and $0.11n^2$ lb ($50n^3$g). The actual values did not scale exactly, but the difference between actual and nominal was less than 2%. Another departure from exact scaling came in the molded thicknessess. It is apparent in Fig. 2 that the thicker specimens have a higher volume fraction of fibers than the thinnest set ($n=1$). This departure from perfect scaling depends on the lay-up and does give rise to problems that are discussed later.

The impactors were dropped from predetermined heights onto the supported beams and the force transmitted to the base measured for all specimens. The output force-time response was recorded on a CRT Visicorder Model 1858. In addition, some of the specimens were instrumented with strain gages located at $0.5n$ in. from the specimen center. The output of these was also recorded on the CRT Visicorder. Ideally, the gages themselves should be scaled but, if the beam deforms in the fundamental quasistatic mode, the gage length factor is eliminated. The strain output was preferred to the force as the output parameter, since the latter tended to be affected by nonscaled natural vibrations in the force transducer. These vibrations had to be filtered out.

Scaling Considerations

The basic outline of the scaling considerations is previously given (for a homogeneous isotropic beam). Using the scaled dimensions described, terms $\pi_1 - \pi_4$ are satisfied (within the limits discussed subsequently as a result of the thickness variations).

On a lamina level, for elastic behavior, the elastic properties E_{11}, E_{22}, ν_{12}, and G_{12} replace E and ν for the isotropic beam. Also, for these laminas the orientations θ_j should be specified. Using the same material (pre-preg) and the lay-ups formed by scaling, the number of plies for each θ_j is sufficient to ensure similarity of elastic behavior in all scale models.

The same material was used for the impactors and the dimensions were scaled so that terms π_7 and π_8 are satisfied. Term π_9 is satisfied by dropping the impactors from the same height so that the impact velocity was held constant for each case.

The term π_{10} means that the strain rate in the smallest beams will be three times greater than in the largest ones. The rate sensitivity of the beams depended on the matrix (in carbon-epoxy laminate) and, thus, on the lay-up. In this study, lay-ups A and B might be expected to be much more rate-sensitive than lay-ups C and D.

It has already been noted that the notch-sensitivity of composites also depends on the lay-up. In this study, lay-up A can fail purely through matrix cracking; that is, a single self-similar damage mechanism. Lay-up B can fail by combined

Table 1 Lay-ups of laminated beams in scale-model tests

Designation	Lay-ups
A_n	$[90_{2(n+1)}]_s$
B_n	$(+45_{n+1}, -45_{n+1})_s$
C_n	$(90_{n+1}, 0_{n+1})_s$
D_n	$(45_n, 90_n - 45_n, 0_n)_s$

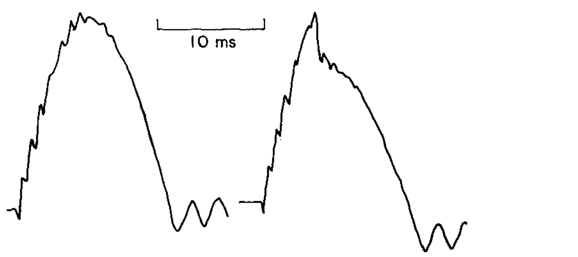
Table 2 Average experimental and theroetical impact duration on small beams, $n=1$

Lay-up	Impact duration, ms	
	Experimental	Theoretical
A	8.95	9.35
B	6.66	6.60
C	6.02	5.93
D	6.50	6.55

matrix cracking and delamination. Lay-up C may, additionally, have fiber fracture as a failure mechanism. Lay-up D requires at least matrix cracking and fiber cracking for failure. Under transverse impact, the first damage occurring in a laminated composite has the form of matrix microcracking.[10] At higher levels of impact the matrix crack will grow until it reaches an interface at which delamination may occur or sufficiently high local stress fields may give rise to fiber fracture in the adjacent ply. The former results in local notch-insensitivity while the latter results in notch sensitivity.[11]

Now consideration should be given to the problem of scaling when damage occurs. The earlier arguments on crack size effects are relevant here. In the scaled laminates, the through-thickness length of a matrix crack is equal to the number of adjoining laminas with the same orientation. Thus, the smaller beams could be stronger than the larger ones, and the apparent strength differences will depend on lay-up.

In summary, elastic scaling is to be expected in the program with departures due to rate-sensitivity effects lay-ups A and B (mainly). These departures are consistent with higher stiffness and smaller duration of impact in smaller specimens. If the failure processes follow the fracture mechanics model, the size effects will be important and strength will not scale. The trend will be for smaller specimens to be stronger.



a) Just before damage b) Just after detectable damage

Fig. 3 Typical strain output traces.

V. Impact Test Results

Typical strain outputs are shown in Fig. 3. The trace in Fig. 3b indicates the occurrence of impact damage. From traces such as these, the duration of impact and the maximum strain (or force) were taken as the output parameters. The duration of the impact should scale with n; that is, the elastic impact durations divided by n should be constant. Figures 4a–4d indicate that this is true to about $\pm 10\%$. With the exception of lay-up B (Fig. 4b), the smallest specimens have the shortest impact duration even after scale factors have been taken into consideration. The apparently anomalous behavior in Fig. 4b led to the discovery that the pre-preg used in this lay-up came from a different, older, batch of material than used in the other specimens. It is not suggested, however, that this is totally responsible for the fact that the durations when $n=1$ are larger than when $n=2$. An elastic analysis (see Appendix) suggests that the impact durations are independent of the impact velocity. This is demonstrated in Fig. 4. Also, departures from this behavior are indicative of the occurrence of impact damage. It is clear that in all cases damage occurs at lower velocities in the larger specimens than in the smaller specimens. This is also illustrated in Fig. 5, which provides a comparison of the drop heights (αV^2) causing initial damage for all specimen types and sizes.

The impact force is expected to scale as n^2. Figures 6a and 6b suggest that this is indeed the case in the elastic regime. However, as previously observed, the small scale models are consistently stronger and carry proportionately higher post-damage loads.

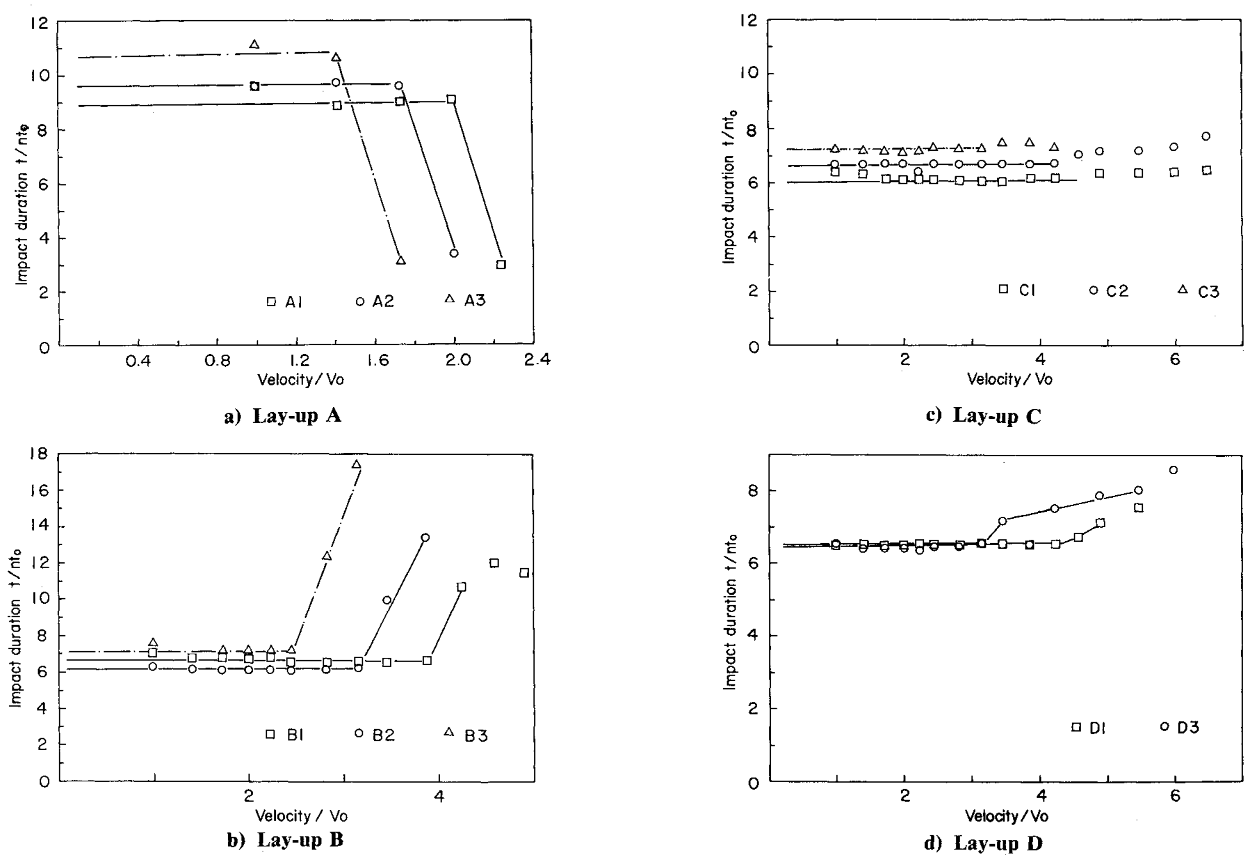


Fig. 4 Scaled normalized impact durations for various normalized impact velocities.

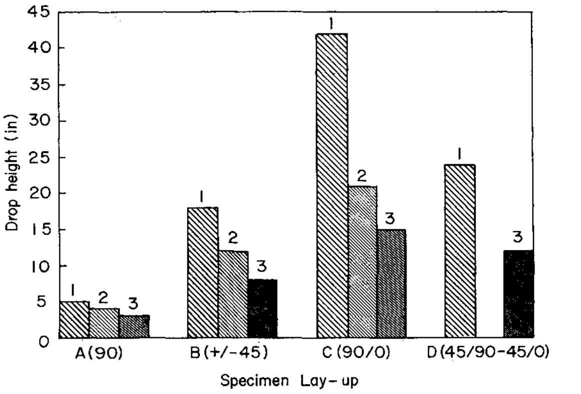


Fig. 5 Impact conditions giving initial impact damage (determined from the strain output traces in the composite beams).

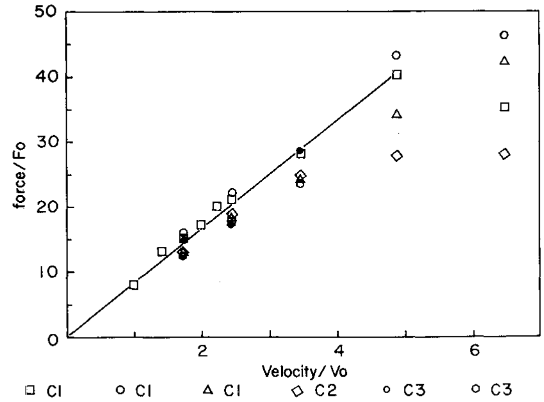
VI. Discussion

The experiments have demonstrated that the impact forces and duration for laminated carbon-epoxy composites are scaled to within about ± 10% provided behavior is elastic. There is evidence that the small scale models are proportionately stiffer than larger scale ones. In the discussions of the similarity conditions, it was mentioned that such effects may occur due to the fundamental scaling problems when rate effects are significant. The results shown in Fig. 4 suggest that an increase of up to 40% in modulus would be required as a consequence of rate effects (assuming that the duration is proportional to $1/\sqrt{E}$). There is, however, no other evidence to support the fact that rate effects are important.

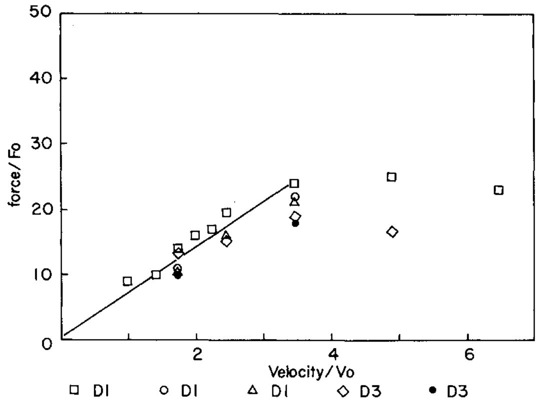
It is unfortunate that the molded thicknesses of the laminates do not scale exactly. In the quasi-isotropic laminates (lay-up D) this is very nearly the case and, for this lay-up at least, elastic scaling is impressive (see Figs. 4d and 6b). However, in the other laminates there appear to be significant thickness effects. It may be argued that the impact duration is inversely proportional to the thickness to the power 1.5 or 2, so that the overall thickness variations, which are of the order of 10%, could be responsible for the observed trends in the scaled impact duration. These happen to act in the same sense as the rate effects, if they occurred, so that it is not possible with these data to separate them.

To clarify this point, slow (quasistatic), three-point bend tests were carried out on the smallest size beam ($n = 1$). The central load was applied through an impactor, and the beams were supported on the rollers used in the impact tests. Load-deflection curves are shown in Fig. 7. From these curves equivalent spring stiffnesses were obtained (assuming linear behavior up to deflections of at least 0.1 in.). The impact durations expected can then be calculated from a simple one-degree-of-freedom model (see Appendix). The comparison of expected and experimental durations is shown in Table 2. The agreement is impressive and suggests that rate effects (in all but lay-up A) are negligible. The fact that the elastic parts of the curves in Figs. 4a-4d do not coincide appears, then, to be due to the departure of scaling in the thickness. As mentioned earlier in the case of lay-up D, thickness was closely scaled and so were the impact durations.

Of greater significance than the strain rate effects here are the size effects on strength and damage. It is clear that the small scale specimens were always stronger than the larger ones. This is probably a microstructural fracture effect as mentioned above. However, there were no microstructural damage studies included in this program. It is then difficult to confirm the fracture mechanics hypotheses in the scaling of composite laminates. Preliminary results from a subsequent study[12] indicate that the absolute size of matrix cracks (in the through-thickness direction) rather than scaled size are important in laminated composites. The tensile strength of small specimens will usually be greater than large ones. One exception to this, however, is when the load bearing (0 deg) fibers are outermost and gripping stress concentrations cause the reverse to be true.[13]



a) Lay-up C



b) Lay-up D

Fig. 6 Scaled normalized impact forces for various impact velocities.

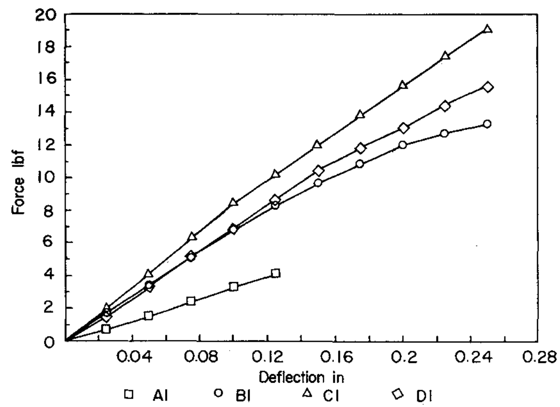


Fig. 7 Load-deflection curves for slow three-point bend tests on composite beams.

Scaling for strength of composite subcomponents is of great practical importance. The results obtained in this program suggest that small scale models may be stronger than prototypes. Then, caution should be exercised in interpreting the results of such tests. Further work is required to identify the micromechanical cause of this effect.

VII. Conclusions

The scaling of laminated fiber composites under impact loading has been considered. Classical scaling laws have been shown to apply for elastic behavior (undamaged) of transversely impacted carbon- fiber reinforced epoxy beams. As expected, the impact duration scaled as the scale factor and the impact force as the scale factor squared. These were true to within about ±10%. Lack of better agreement was shown to be due to thicknesses not being scaled exactly even though the number of plies were.

Rate effects were insignificant in the lay-ups and material system tested. The only detectable rate effect occurred in lay-up A in which the flexural stiffness was matrix dominated.

Important size effects were noted as far as strength was concerned. Smaller specimens were always stronger than larger ones. This was thought to be due to the absolute size of matrix cracks and their effect on subsequent damage characterstics.

Appendix

Analytical Considerations

Shivakumar et al.[14] have provided analytical expressions for the impact force and duration of circular, isotropic laminates. The analysis is elastic but includes some nonlinear terms. In fact, two analyses are presented. These are quasistatic and dynamic. Both are used subsequently to obtain nondimensional expressions that illustrate the nonuniqueness of Pi-terms. They were also adapted for elastic analyses of the beams impacted in the present study. Bending stiffnesses were calculated from laminate theory and/or obtained by experiment.

a) Quasistatic Analysis

The maximum impact force is given by solution to the equations

$$\bar{p} = \bar{\omega}(\alpha + \beta\bar{\omega}^2)$$

$$\bar{e} = \alpha\bar{\omega}^2 + \beta\omega^{=4} + 0.8[(\alpha\bar{\omega} + \beta\bar{\omega}^3)^5/\gamma]^{1/3}$$

$$\bar{a} = [\bar{p}\bar{r}^{3/2}/\gamma^{1/2}]^{1/3}$$

These are valid if $\bar{\omega} < 0.4$. The nondimensional terms are

$$\bar{p} = F/Eh^2 \qquad \bar{\omega} = \omega/h$$

$$\bar{e} = mV^2/Eh^3 \qquad \alpha = K_{bs}E_x h$$

$$\bar{a} = a_c/h \qquad \beta = K_m h/E_x$$

$$\gamma = n^2/E_x^2 h \qquad \bar{r} = R/h$$

where K_{bs} is the bending/shear stiffness, K_m a membrane stiffness, a_c the constact radius, n the Hertzian indentation parameter, and E_x the laminate beam axial modulus.

b) Dynamic Analysis

The two-degree-of-freedom model has solutions given by

$$\ddot{\bar{x}}_1 + \lambda\theta\,|(\bar{x}_1 - \bar{x}_2)|^{1.5} = 0$$

$$\ddot{\bar{x}}_2 + \eta\bar{x}_2 + \xi\bar{x}_2 - \lambda\mu\,|(\bar{x}_1 - \bar{x}_2)|^{1.5} = 0$$

$$\bar{p} = \bar{x}_2(\alpha + \beta\bar{x}_2^2)$$

where

$$\bar{x}_2(t) = \frac{\omega}{h}(t), \qquad \frac{x_1}{h}(t)$$

is the indentation deflection.

These expressions were used to design the beams such that indentation effects could be neglected. The impact masses were also chosen to be much larger than the beam mass. The one-degree-of-freedom analysis is then readily obtained. The boundary conditions used were

$$\bar{x}_1(t) = \bar{x}_2(0) = \dot{\bar{x}}_2(0) = 0$$

$$\dot{\bar{x}}_1(0) = (v/c)$$

where $c^2 = (E_x/\rho)$, and ρ is the mass density.

The nondimensional parameters were

$$\bar{p} \qquad \eta = \alpha(h^2/\ell^2)$$

$$\bar{x}_1 = x_1/h \qquad \xi = \beta(h^2/\ell^2)$$

$$\bar{x}_2 = x_2/h = \omega/h \qquad \mu = \gamma^{1/2}(h^2/\ell^2)$$

$$t^{-2} = t/t_0 \qquad \theta = (m/m_i)\gamma^{1/2}(h^2/\ell^2)$$

$$t_0 = h/c$$

Here there are nine nondimensional groups and in the quasistatic analysis there were eight. The difference is, of course, in the introduction of time in the dynamic analysis. It is easily seen that the groups are multiples of each other.

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